# A Morris-Shin Model with Proper Priors

In this appendix, we derive equilibrium choice strategies for the private signals treatments. We study the case where the state is distributed uniformly on a the interval $[0, l]$, and the signals are conditionally normally distributed for the private signals treatments. Let $f(s|\theta)$ denote the density of a signal conditional on the realized state $\theta$, and let $F(s|\theta)$ its associated cumulative distribution function.

As we will show, equilibrium consists of playing a strategy identical to the improper priors framework plus an additional term correcting for the prior. This latter term is highly non-linear but small except at the edges of the distribution. Thus, for intermediate signal values, the improper priors model is an excellent approximation to the fully Bayesian model. An important implication of this analysis is to show that the degree of center-bias we see in the data far exceeds what can be explained by placing weight on priors. We omit the case where there is a public signal since the nonlinearities in the equilibrating process make this analytically intractable.

**One Private Signal**

When individuals each receive a single private signal, using arguments identical to those in Morris and Shin, the unique equilibrium consists of each agent $i$ choosing the action

$$a_i = E[\theta|s_i]$$

Thus, we need only compute $E[\theta|s_i]$. Using Bayes' rule, this amounts to

$$E[\theta|s_i] = \frac{\int_0^l \theta f(s_i|\theta) \frac{1}{l} d\theta}{\int_0^l f(s_i|t) \frac{1}{l} dt}$$

Next, recall that $s_1 = \theta + \varepsilon$ where $\varepsilon$ is normally distributed. Hence

$$\begin{aligned} f(s_i|\theta) &= \Pr(\theta + \varepsilon_1|\theta) = s_i \\ &= \Pr(\varepsilon_1 = s_i - \theta) \\ &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i - \theta}{\sigma}\right)^2} \end{aligned}$$

The difference in the cdf evaluated at $s_1$ and $s_1 - l$ is proportional to

$$\psi = \operatorname{erf}\left(\frac{s_i}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{s_i - l}{\sqrt{2}\sigma}\right)$$

where $\operatorname{erf}(\cdot)$ is the error function.

And so the denominator becomes

$$\begin{aligned}\int_0^l f(s_i|t)\,dt &= \int_0^l \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i-t}{\sigma}\right)^2} dt \\ &= \frac{1}{2}\psi\end{aligned}$$

Now, substituting, we obtain

$$\begin{aligned}E[\theta|s_i] &= \int_0^l \theta f(\theta|s_i)\,d\theta \\ &= \frac{1}{\psi}\int_0^l \theta \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i-\theta}{\sigma}\right)^2} d\theta \\ &= \frac{1}{\psi\sqrt{\pi}} \times \left(\begin{array}{c} -\sigma\sqrt{2}\exp\left(-\frac{1}{2}\frac{(-l+s_i)^2}{\sigma^2}\right) - s_i \operatorname{erf}\left(\frac{1}{2}\sqrt{2}\frac{-l+s_i}{\sigma}\right)\sqrt{\pi} \\ +\sigma\sqrt{2}e^{-\frac{1}{2}\frac{(s_i)^2}{\sigma^2}} + s_1 \operatorname{erf}\left(\frac{1}{2}\frac{s_i}{\sigma}\sqrt{2}\right)\sqrt{\pi} \end{array}\right) \\ &= \frac{s_i\sqrt{\pi}\left(\operatorname{erf}\left(\frac{1}{2}\frac{s_i}{\sigma}\sqrt{2}\right) - \operatorname{erf}\left(\frac{1}{2}\sqrt{2}\frac{-l+s_i}{\sigma}\right)\right)}{\psi\sqrt{\pi}} + \frac{\sigma\sqrt{2}\left(e^{-\frac{1}{2}\frac{(s_i)^2}{\sigma^2}} - \exp\left(-\frac{1}{2}\frac{(-l+s_i)^2}{\sigma^2}\right)\right)}{\psi\sqrt{\pi}} \\ &= s_i + \frac{\sigma\sqrt{2}\left(e^{-\frac{1}{2}\frac{(s_i)^2}{\sigma^2}} - \exp\left(-\frac{1}{2}\frac{(-l+s_i)^2}{\sigma^2}\right)\right)}{\psi\sqrt{\pi}}\end{aligned}$$

Using arguments identical to those for the improper priors case, it may be readily shown that:

**Proposition 1** *In the Morris-Shin model with proper priors and a state that is uniformly distributed on* $[0, l]$, *the unique equilibrium consists of all individuals choosing action* $a(s)$ *where*

$$a(s) = E[\theta|s] = s + \frac{\sigma\sqrt{2}\left(e^{-\frac{1}{2}\frac{(s)^2}{\sigma^2}} - \exp\left(-\frac{1}{2}\frac{(-l+s)^2}{\sigma^2}\right)\right)}{\psi\sqrt{\pi}}$$

Figure 3 plots the equilibrium strategy as a function of the signal, $s$ for the parameter values used in the experiment. $\sigma = 833$, $l = 10,000$. The solid line denotes the equilibrium strategy with proper priors; the dashed line denotes the equilibrium strategy under improper priors.

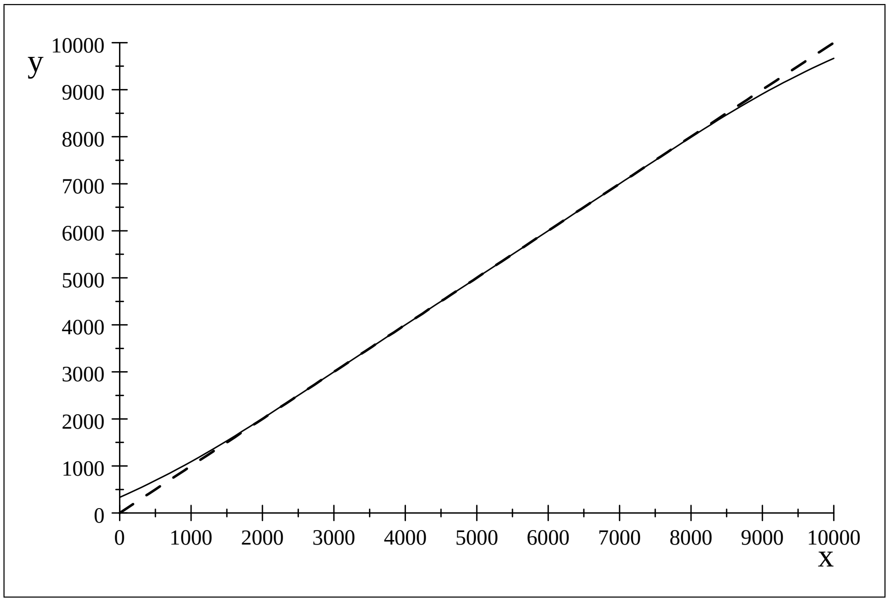


**Figure 3: Optimal choice (y) conditional on a single private signal (x)**

As the figure shows, the improper priors model is a good approximation for a fully Bayesian specification for signals away from the edges of the distribution. Moreover, the amount of center bias we observe in the data is inconsistent with optimal Bayesian weighting of priors.

**Two Private Signals**

Next we study the two signal case. The object of interest is

$$E\left[\theta | s_i^1, s_i^2\right] = \int_0^l \theta f\left(\theta | s_i^1, s_i^2\right) d\theta$$

Now, let us use Bayes' rule to determine $f\left(\theta | s_i^1, s_i^2\right)$.

$$f\left(\theta | s_i^1, s_i^2\right) = \frac{f\left(s_i^1, s_i^2 | \theta\right) g\left(\theta\right)}{\int_0^1 f\left(s_i^1, s_i^2 | t\right) g\left(t\right) dt}$$

Recall that $g\left(\theta\right) = \frac{1}{l}$ and signals are conditionally independent so

$$f\left(\theta | s_i^1, s_i^2\right) = \frac{f_1\left(s_i^1 | \theta\right) f_2\left(s_i^2 | \theta\right)}{\int_0^l f_1\left(s_i^1 | t\right) f_2\left(s_i^2 | t\right) dt}$$

Next, recall that $s_i^1 = \theta + \varepsilon$ where $\varepsilon$ is normally distributed. Hence

$$\begin{aligned} f_1\left(s_i^1|\theta\right) &= \Pr\left(\theta+\varepsilon_1|\theta\right) = s_i^1 \\ &= \Pr\left(\varepsilon_1 = s_i^1 - \theta\right) \\ &= \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i^1-\theta}{\sigma_1}\right)^2} \end{aligned}$$

and similarly for $f_2\left(s_i^2|\theta\right)$.

And so the denominator becomes

$$\int_0^l f_1\left(s_i^1|t\right) dt = \int_0^l \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i^1-t}{\sigma_1}\right)^2} \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i^2-t}{\sigma_2}\right)^2} dt$$

And we obtain

$$\int_0^l f_1\left(s_i^1|t\right) dt = \frac{1}{4}\zeta$$

where $\zeta$ is the two signal analog to the expression $\psi$ given above. Formally,

$$\zeta = \sqrt{2}\exp\left(-\frac{1}{2}\frac{\left(s_i^1-s_i^2\right)^2}{\sigma_2^2+\sigma_1^2}\right)\frac{\operatorname{erf}\left(\frac{s_i^1\sigma_2^2+s_i^2\sigma_1^2}{\sqrt{2}\sigma_1\sigma_2\sqrt{\sigma_2^2+\sigma_1^2}}\right)-\operatorname{erf}\left(\frac{\left(s_i^1-l\right)\sigma_2^2+\left(s_i^2-l\right)\sigma_1^2}{\sqrt{2}\sigma_1\sigma_2\sqrt{\sigma_2^2+\sigma_1^2}}\right)}{\sqrt{\pi\left(\sigma_2^2+\sigma_1^2\right)}}$$

Now, substituting, we obtain (after much simplification)

$$\begin{aligned} E\left[\theta|s_i^1,s_i^2\right] &= \int_0^l \theta f_1\left(\theta|s_i^1\right) f_2\left(\theta|s_i^1\right) d\theta \\ &= \frac{1}{\frac{1}{4}\zeta}\int_0^l \theta\frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i^1-\theta}{\sigma_1}\right)^2}\frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s_i^2-\theta}{\sigma_2}\right)^2} d\theta \\ &= \frac{\sigma_2^2 s_i^1+\sigma_1^2 s_i^2}{\sigma_2^2+\sigma_1^2} \\ &\quad + \frac{2\sigma_1\sigma_2\left(\exp\left(-\frac{1}{2}\frac{s_i^1\sigma_2^2+s_i^2\sigma_1^2}{\sigma_1^2\sigma_2^2}\right)-\exp\left(-\frac{1}{2}\frac{\sigma_1^2\left(s_i^2-l\right)^2+\sigma_2^2\left(-l+s_i^1\right)^2}{\sigma_1^2\sigma_2^2}\right)\right)}{\psi\pi\left(\sigma_2^2+\sigma_1^2\right)} \end{aligned}$$

And again, using arguments identical to the improper priors case, it follows that:

**Proposition 2** *In the Morris-Shin model with proper priors and a state that is uniformly* *distributed on* $[0, l]$ *, the unique equilibrium consists of all individuals choosing action* $a\left(s_i^1, s_i^2\right)$

$$\begin{aligned} a\left(s_i^1, s_i^2\right) &= E\left[\theta | s_i^1, s_i^2\right] = \frac{\sigma_2^2 s_i^1 + \sigma_1^2 s_i^2}{\sigma_2^2 + \sigma_1^2} \\ &+ \frac{2\sigma_1\sigma_2\left(\exp\left(-\frac{1}{2}\frac{s_i^1\sigma_2^2 + s_i^2\sigma_1^2}{\sigma_1^2\sigma_2^2}\right) - \exp\left(-\frac{1}{2}\frac{\sigma_1^2\left(s_i^2 - l\right)^2 + \sigma_2^2\left(-l + s_i^1\right)^2}{\sigma_1^2\sigma_2^2}\right)\right)}{\zeta\pi\left(\sigma_2^2 + \sigma_1^2\right)} \end{aligned}$$

It may be readily shown that away from the edges of the distribution, the equilibrium actions are approximately equal to the weighted sample mean of the signals, $\frac{\sigma_2^2 s_i^1 + \sigma_1^2 s_i^2}{\sigma_2^2 + \sigma_1^2}$. Since $E\left[\theta | s_i^1, s_i^2\right]$ is a highly nonlinear function of the signals, incorporating this expectation into the calculation of a fixed point, as is required for the determination of equilibrium when $s^2$ is public, resists analytic solution.

# B Reconciling our loss function with that of Morris/Shin

Morris and Shin define the utility function as

$$\begin{aligned} u_i &= -\left(1 - r\right)\left(a_i - \theta\right)^2 - r\left(L_i - \overline{L}\right) \\ L_i &= \frac{1}{n}\sum_j \left(a_j - a_i\right)^2 \\ \overline{L} &= \frac{1}{n}\sum_j L_j \end{aligned}$$

The second term calculates the individual's loss as a function of all individuals' actions. Our protocol instructs participants that the loss is a function of the average of all individuals' actions. As we shall show, these are equivalent. The term of interest is $L_i - \overline{L}$.

$$
\begin{aligned}
L_i - \overline{L} &= L_i - \frac{1}{n}\sum_j L_j \\
&= \frac{n-1}{n} L_i - \frac{1}{n}\sum_{j\neq i} L_j \\
&= \frac{n-1}{n^2}\sum_j (a_j - a_i)^2 - \frac{1}{n^2}\sum_{j\neq i}\sum_k (a_j - a_k)^2 \\
&= \frac{n-1}{n^2}\sum_{j\neq i} (a_j - a_i)^2 - \frac{1}{n^2}\sum_{j\neq i}\left[(a_j - a_i)^2 + \sum_{k\neq i}(a_j - a_k)^2\right] \\
&= \frac{n-2}{n^2}\sum_{j\neq i} (a_j - a_i)^2 - \frac{1}{n^2}\sum_{j\neq i}\left[\sum_{k\neq i}(a_j - a_k)^2\right] \\
\frac{d\left(L_i - \overline{L}\right)}{da_i} &= \frac{2\,(n-2)}{n^2}\sum_{j\neq i}(a_i - a_j) \\
&= \frac{2\,(n-2)}{n^2}\left((n-1)\,a_i - \sum_{j\neq i} a_j\right) \\
&= \frac{2\,(n-2)}{n^2}\left((n-1)\,a_i - (n-1)\,\overline{a}_{-i}\right) \\
&= \frac{2\,(n-2)\,(n-1)}{n^2}\left(a_i - \overline{a}_{-i}\right)
\end{aligned}
$$

We define the utility function in a slightly different way.

$$
\begin{aligned}
u_i &= -(1-r)\,(a_i - \theta)^2 - r\left(L_i - \overline{L}\right) \\
L_i &= (a_i - \overline{a})^2 \\
\overline{L} &= \frac{1}{n}\sum_j L_j
\end{aligned}
$$

Again, the term of interest is $L_i - \overline{L}$.

$$
\begin{aligned}
L_i - \overline{L} &= (a_i - \overline{a})^2 - \frac{1}{n}\sum_j (a_j - \overline{a})^2 \\
&= \frac{(n-1)}{n}(a_i - \overline{a})^2 - \frac{1}{n}\sum_{j\neq i}(a_j - \overline{a})^2 \\
&= \frac{(n-1)}{n}\left(a_i - \frac{1}{n}a_i - \frac{1}{n}\sum_{k\neq i} a_k\right)^2 - \frac{1}{n}\sum_{j\neq i}\left(a_j - \frac{1}{n}a_i - \frac{1}{n}\sum_{k\neq i} a_k\right)^2 \\
&= \frac{(n-1)}{n}\left(\frac{(n-1)}{n}a_i - \frac{(n-1)}{n}\overline{a}_{-i}\right)^2 - \frac{1}{n}\sum_{j\neq i}\left(a_j - \frac{1}{n}a_i - \frac{(n-1)}{n}\overline{a}_{-i}\right)^2 \\
&= \frac{(n-1)^3}{n^3}(a_i - \overline{a}_{-i})^2 - \frac{1}{n^3}\sum_{j\neq i}(na_j - a_i - (n-1)\,\overline{a}_{-i})^2 \\
\frac{d\left(L_i - \overline{L}\right)}{da_i} &= \frac{2\,(n-1)^3}{n^3}(a_i - \overline{a}_{-i}) + \frac{2}{n^3}\sum_{j\neq i}(na_j - a_i - (n-1)\,\overline{a}_{-i}) \\
&= \frac{2\,(n-1)^3}{n^3}(a_i - \overline{a}_{-i}) + \frac{2}{n^3}\left(n\sum_{j\neq i} a_j - (n-1)\,a_i - (n-1)^2\,\overline{a}_{-i}\right) \\
&= \frac{2\,(n-1)^3}{n^3}(a_i - \overline{a}_{-i}) + \frac{2}{n^3}\left(n\,(n-1)\,\overline{a}_{-i} - (n-1)\,a_i - (n-1)^2\,\overline{a}_{-i}\right) \\
&= \frac{2\,(n-1)^3}{n^3}(a_i - \overline{a}_{-i}) + \frac{2\,(n-1)}{n^3}(n\overline{a}_{-i} - a_i - (n-1)\,\overline{a}_{-i}) \\
&= \frac{2\,(n-1)^3}{n^3}(a_i - \overline{a}_{-i}) - \frac{2\,(n-1)}{n^3}(a_i - \overline{a}_{-i}) \\
&= \frac{2\,(n-1)}{n^3}(a_i - \overline{a}_{-i})\left[(n-1)^2 - 1\right] \\
&= \frac{2\,(n-1)}{n^3}(a_i - \overline{a}_{-i})\left[n^2 - 2n\right] \\
&= \frac{2\,(n-1)\,(n-2)}{n^2}(a_i - \overline{a}_{-i})
\end{aligned}
$$

From here the derivation proceeds as in Morris & Shin.

# C Instructions for Experiment

Decision-Making Study Instructions

General Rules

This is an experiment in the economics of decision-making. If you follow the instructions carefully and make good decisions you can earn a considerable amount of money. You will be paid in private and in cash at the end of the session.

There are up to fifteen people participating in this session. It is important that you do not talk, or in any way attempt to communicate, with other subjects during the session. It is also important that you do not look at other subjects' computer screens at any time. If you have a question, raise your hand and a monitor will come over to where you are sitting and answer your question privately.

The experiment will consist of a number of rounds. In each round, you will have the opportunity to earn points. At the end of the session, you will be paid according to the number of points you earned throughout the experiment.

Description of Each Round

At the beginning of each round, you will see a slider bar on the computer screen. Your task in each round is to choose a point on the slider bar, using the information presented to you as you see fit.

In each round, there is a Secret Spot, which could be any point on the slider bar. The Secret Spot is the same point for all players in any particular round, though it will change from round to round. You will not be informed as to the exact location of the Secret Spot. However, in each round, a black arrow will appear somewhere above the slider bar. This arrow is your Private Hint of the location of the Secret Spot. Your Hint will not, generally, give the exact location of the Secret Spot. The Hint is equally likely to be to the left or to the right of the Secret Spot, and will usually be close. Above the slider bar is a series of tick marks. There is approximately a 75% chance that the Secret Spot will be within one tick mark's distance to either side of your Private Hint. There is approximately a 95% chance that the Secret Spot will be within two tick marks' distance to either side of your Private Hint. Your Private Hint is yours alone; all of the other subjects will get their own Private Hints, which will be different from yours. (The Secret Spot is the same for all subjects).

In some rounds, a blue arrow will appear above the slider bar in addition to the black arrow. This blue arrow is a Public Hint. If you receive a Public Hint, all subjects are also receiving the same Public Hint. The location of the Public Hint is the same for all subjects, though it may change from round to round. The Public Hint, like your Private Hint, will not generally give the exact location of the Secret Spot. It follows slightly different rules regarding its accuracy: there is a 75% chance that the Secret Spot will be within *two* tick marks' distance to either side of the Public Hint, and there is a 95% chance that the Secret Spot will be within *four* tick marks' distance to either side of the Public Hint. In other words, the Public Hint is half as accurate, as your Private Hint, on average.

There are several ways to move the indicator on the slider bar. You can click on the indicator and drag it left or right to the position you choose. You can also click on any point on the slider bar, and the indicator will move to that location. When you are satisfied with the position of the indicator on the slider bar, enter your choice by clicking the Enter button or pressing the Enter key on your keyboard.

After all subjects have made their decisions, the results will appear on your screen. Underneath the slider bar will appear two more arrows. The red arrow indicates the location of the Secret Spot. The green arrow indicates the average of all subjects' choices for that round. Also, you will now be shown the choices of all the other subjects, in random order. You will see for each other subject the location of her Private Hint, the location of the Public Hint (if shown), the location of their choice, the Secret Spot, and the average of all subjects' choices. You will also see your points earned for the round. When you are ready to proceed to the next round, hit the Enter key on the keyboard or click the Next Round button to move on.

Earning Points

You begin each round with 1000 points. You can then gain or lose points, depending upon the location of your choice, the Secret Spot, and the locations of the other subjects' choices.

a) You lose points according to the square of the distance between the location you choose and the Secret Spot. Thus, if your guess is close to the Secret Spot, you will lose fewer points than if your guess is far from the Secret Spot.
b) You will earn points if your guess is closer to the average of all subjects' guesses than most other players. You will lose points if your guess is farther from the average than most other subjects' guesses.

In summary, you will earn the most possible points if your guess is close to the Secret Spot and also close to the average guess. Getting close to the average guess is four times as important as getting close to the Secret Spot. Specifically, your points for the round are determined by the following formula:

$$Score = 1000 - (guess - SecretSpot)^2 - 4(loss - avgloss) \text{ where}$$

$$loss = (guess - averageguess)^2 \text{ and } avgloss \text{ is the average of } loss \text{ for all subjects.}$$

Each round, your total net points earned will be displayed. Also, a running total of your points earned during the experiment will be displayed on the screen. If your choice is particularly bad or unlucky, it is entirely possible that you will lose points in any particular round.

After all rounds have been completed, your computer will display the total points you accumulated and the amount of money you have earned. At the end of the experiment, you will be paid according to the number of points that you accumulated throughout all rounds of the experiment.

Table A1: Testing Effects of Changes in Population Size

| | Specification | | | |
|---|---|---|---|---|
| | A | B | C | D |
| More precise (private) signal | 0.532 | 0.532 | 0.546 | 0.546 |
| | (0.066) | (0.066) | (0.066) | (0.066) |
| Less precise (public) signal | 0.371 | 0.371 | 0.362 | 0.362 |
| | (0.069) | (0.069) | (0.072) | (0.072) |
| n (Number of participants in a session) | -3.08 | -3.089 | 1.483 | 1.475 |
| | (13.457) | (13.462) | (12.427) | (12.431) |
| More precise (private) signal * n | 0.004 | 0.004 | 0.004 | 0.004 |
| | (0.005) | (0.005) | (0.005) | (0.005) |
| Less precise (public) signal * n | -0.002 | -0.002 | -0.003 | -0.003 |
| | (0.005) | (0.005) | (0.005) | (0.005) |
| Center bias | 0.093 | 0.093 | 0.088 | 0.088 |
| | (0.038) | (0.038) | (0.034) | (0.034) |
| Individual Random Effects | No | Yes | No | Yes |
| Demographic variables | No | No | Yes | Yes |
| $R^2$ | 0.96 | 0.96 | 0.96 | 0.96 |
| Number of observations | 2340 | 2340 | 2340 | 2340 |

Notes: This table examines whether choice weights on signals change with the population size in the public signal treatment. Coefficients on signals are significant at 1% level in all specifications. Coefficients on n and interactions are not significant in any specification.

Standard errors clustered by participant.

Table A2: Testing in Truncated Signal Space

| | Specification | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Private signal | 0.881 | 0.888 | 0.881 | 0.892 |
| | (0.007) | (0.014) | (0.009) | (0.018) |
| Center bias | 0.120 | 0.112 | 0.121 | 0.110 |
| | (0.007) | (0.013) | (0.009) | (0.017) |
| Individual Random Effects | Yes | Yes | Yes | Yes |
| Demographic variables | No | No | Yes | Yes |
| Truncated (2000<signal<8000) | No | Yes | No | Yes |
| $R^2$ | 0.96 | 0.89 | 0.96 | 0.89 |
| Number of observations | 1920 | 1264 | 1920 | 1264 |

Notes: This table examines the effect of truncating signal realizations near the endpoints of the state space from the data. Columns B and D drop all choice data where the signal realization is less than 2000 or more than 8000. Demographic variables include gender, categorical variables indicating a major in economics or mathematics, the number of college-level mathematics courses taken and the number of college-level economics courses taken. These variables are interacted with the signal realization.

Standard errors clustered by participant.